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QUASIDYNAMIC MODELING OF HEAT-TRANSFER PROCESSES

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An analytic method is developed for describing heat-transfer dynamics, making effective use of the quasi-dynamic features of the processes. The results are given specific form for heat exchangers of two-flow, one-flow, and immersion type.

Simulative modeling of transient conditions in different technological systems is closely related to mathematical description of the dynamic changes in parameters characterizing the structural elements of such systems. It is expedient here to distinguish two classes of nonsteady processes differing in their rates: 1) fast (pulsed) processes, in which the characteristic times τ^* of parameter variation at the input to the element are commensurate with, or less then, the relaxational times τ_r of this element ($\tau^* \lesssim \tau_r$); 2) slow processes, for which $\tau^* \gg \tau_r$; henceforward, these latter processes will be referred to as quasidynamic. Often, the transition from one set of operating conditions of the system to another may be divided into two analogous stages. The quasidynamic stage is the principal component of transient processes in many complex engineering systems, which explains the increased interest in the creation of corresponding models.

Note that most traditional methods of describing the dynamics of heat-transfer processes (numerical methods, Laplacian schemes, etc.) poorly reflect, and make practically no use of, the features of slow evolution of the systems, which entails incorrect computer analysis and complicates the solution of the problem of controlling processes in real-time conditions. This leads to the need to develop high-speed analytical models taking account of the quasi-dynamic features of the processes which occur. The creation of such models (for the description of heat-transfer elements of cryogenic systems, in the present case) is the aim of the present work.

The basic features of the theory developed are clearly exhibited in the simplest model of dynamic heat transfer between an isothermal wall and a one-dimensional heat-carrier flow, when the evolution of the flow temperature T is described by the classical relation

$$\rho\Omega C_p \frac{\partial T}{\partial \tau} + GC_p \frac{\partial T}{\partial x} = \alpha\Pi (T_w - T).$$

Then, introducing the transport time $\tau_0 = \rho\Omega/G$ and the interaction parameter $U = \alpha\pi/(GC_p)$, the equation obtained is

$$\tau_0 \frac{\partial T}{\partial \tau} + \frac{\partial T}{\partial x} = U(T_w - T), \quad 0 < x \leqslant 1, \quad \tau > 0, \tag{1}$$

with the boundary conditions: $T(0, \tau) = T_1(\tau)$, $T(x, 0) = T_0(x)$. Assuming, for the sake of simplicity, that τ_0, U, T_W = const, it may readily be shown that, at time τ exceeding the transport time τ_0, the output temperature $T_2(\tau) = T(1, \tau)$ is determined not by the initial distribution $T_0(x)$ but by the inlet temperature T_1 as a function of a delayed argument

$$T_2(\tau) = T_w + \exp(-U)[T_1(\tau - \tau_0) - T_w]. \tag{2}$$

Writing the analytical solution of Eq. (1) in the form in Eq. (2) offers the possibility of dispensing with the traditional numerical integration of Eq. (1). This form of solution removes the many problems intrinsic to the numerical method, fundamentally increases

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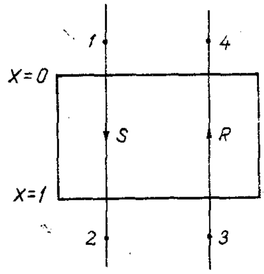

Fig. 1. Indexing of boundary points and conventional notation for two-flow heat exchanger.

the speed of the calculations, and permits the organization of a simple and reliable algorithm for the description of the evolution of the parameters of systems containing similar structural elements. If real heat-transfer processes permitted elementary solutions of the type in Eq. (2), most problems in simulative modeling would present no fundamental difficulties. Thus, there arises a very urgent question: is there a sufficiently broad class of processes whose features allow elementary solutions expressed in terms of delayed arguments to be obtained? The results below clearly demonstrate that the quasidynamic processes have the required features; the importance of investigating these processes has already been noted.

The essence of the quasidynamic description and its basic stages may be formulated as follows.

1. The analytical solution of the static problem (when $\partial/\partial\tau = 0$) is sought. The vector **Y** of steady parameter values at the output from the element and the corresponding coordinate distributions as a function of the input-parameter vector **X** are determined here.

2. The corrections to **Y** associated with the dynamic components of the equations of the process are determined. It is assumed that these components are much less than the characteristic static terms, i.e., $\tau_0\partial/\partial\tau \sim \tau_0/\tau^* \ll 1$. Then, according to perturbation theory, the static profiles obtained may be used in calculating these corrections with in the framework of the first approximation.

3. Approximate values of the delay times are derived from the coefficients of the dynamic derivatives and the parameters of the static dependences; the static functions of the delayed arguments may then be interpreted as quasidynamic solutions. In particular, for a smooth one-parameter dependence, this transition may be written in the form

$$Y(\tau) = f[X(\tau)] - \gamma\dot{X} \approx f\left[X\left(\tau - \frac{\gamma}{f'_X}\right)\right]. \quad (3)$$

This relation (with a delayed argument) is more correct than the expression for Y in terms of the derivative $\dot{X}$, since it allows the influence of the form of the input parameters to be reflected in explicit form and also permits significant simplification and standardization of the algorithm for the composition of systems including such elements. The use of delayed arguments is especially beneficial if the input parameter X changes discontinuously (for example, on switching on any external couplings), when the quasidynamic term $\gamma\dot{X}$ is not small. However, when using Eq. (3), caution is necessary in the case of a weak dependence of the static solution on the input parameter X when $f_X' \to 0$.

Thus, quasidynamic description entails determining the static functions f(X), their derivatives f_X', and the coefficients γ. As an example, it is expedient to consider the application of the given scheme to the simplest equations of the process in Eq. (1). Then, in the first stage, the solution of the steady problem with a parametric dependence on τ

$$\frac{d\tilde{T}}{dx} = U(T_w - \tilde{T}), \quad \tilde{T}|_{x=0} = T_1(\tau)$$

takes the form

$$\tilde{T}(x, \tau) = T_w + \exp(-Ux)[T_1(\tau) - T_w].$$

The second stage gives

$$T_2(\tau) = \tilde{T}(1, \tau) - \tau_0 \int_0^1 \exp[U(x-1)] \frac{\partial\tilde{T}(x, \tau)}{\partial\tau} dx = T_w + \exp(-U)[T_1(\tau) - T_w] - \tau_0 \exp(-U)\dot{T}_1.$$

Finally, assuming that

$$Y = T_2, \quad X = T_1, \quad f'_X = \exp(-U), \quad \gamma = \tau_0 \exp(-U),$$

in the third stage

$$T_2(\tau) \approx T_w + \exp(-U)\,[T_1(\tau - \tau_0) - T_w].$$

This example, the simplest, leads to absolute coincidence of the quasidynamic approximation and the accurate analytical solution in Eq. (2), which, of course, cannot be expected for more complex initial systems.

In the case of several variables, the dependence of the output parameters Y_i on the input parameters X_j is given by an obvious generalization of Eq. (3)

$$Y_i(\tau) = f_i[X_j(\tau)] - \sum_j \gamma_{ij}\dot{X}_j \approx f_i[X_j(\tau - \tau_{ij})], \tag{3a}$$

where

$$\tau_{ij} = \gamma_{ij} \Big/ \left(\frac{\partial f_i}{\partial X_j}\right).$$

In writing the quasidynamic algorithm, it may be useful to express only the dynamic deviations in terms of the delayed arguments rather than the static function f(**X**). Then, in the same approximation, the equivalent form is

$$Y_i(\tau) \approx f_i[X_j(\tau)] - \sum_j \frac{\partial f_i}{\partial X_j}[X_j(\tau) - X_j(\tau - \tau_{ij})]. \tag{3b}$$

If the time τ_{ij} is found to be negative for some arguments, the corresponding term in the sum in Eq. (3b) is expediently replaced, with the same accuracy, by the term

$$\frac{\partial f_i}{\partial X_j}[X_j(\tau - |\tau_{ij}|) - X_j(\tau)].$$

These relations are now given specific form for the description of quasidynamic processes in the most inertial elements of cryogenic equipment: two-flow, single-flow, and immersion heat-exchangers. The static dependence of the input temperatures on the output temperatures was investigated for this case in a series of works [1-5], where the characteristic parameter ranges for which these dependences are significantly nonlinear in form were also determined [4, 5]. However, quasidynamic corrections are expediently calculated within the framework of a linear model, since they are presumed to be small. The averaging of the coefficients of the initial equations required for this purpose should ensure coincidence of the output temperatures calculated from linear and nonlinear models; this is assumed below. Such problems were discussed in [1-3], where combinatorial problems of the search for an arbitrary pair of boundary temperatures were solved. However, this tendency to generality fundamentally increases the unwieldiness of the final formulas. Therefore, in the present work, specific form is given to the most important version of the quasidynamic search for output temperatures, which allows the final results to be expressed in the form of time-delayed arguments (the expediency of their use was discussed above).

Of most importance, in terms of applications, is the quasidynamic description of heat-transfer processes through the wall in a two-flow counterflow heat exchanger. Quasidynamic theory is now outlined in detail for this case. The corresponding traditional equations of the evolution of temperature fields may be written (Fig. 1) in the form

$$\begin{aligned}
&G_S C_p^S \frac{\partial T_S}{\partial x} - \alpha_S \Pi_S (T_w - T_S) = -q_S = -\left(\rho_S \Omega_S C_p^S \frac{\partial T_S}{\partial \tau} + \hat{q}_S\right),\\
&G_R C_p^R \frac{\partial T_R}{\partial x} + \alpha_R \Pi_R (T_w - T_R) = q_R = \rho_R \Omega_R C_p^R \frac{\partial T_R}{\partial \tau} + \hat{q}_R,\\
&\alpha_S \Pi_S (T_S - T_w) + \alpha_R \Pi_R (T_R - T_w) = q_w = \rho_w \Omega_w C_w \frac{\partial T_w}{\partial \tau} + \hat{q}_w.
\end{aligned} \tag{4}$$

The influence of the hydraulic losses, heat conduction, external heat supply, and so on is assumed to be small here. The corresponding terms symbolically included in $\hat{q}$ may be given specific form after detailed thermohydrodynamic description. Assuming also that

$$k\Pi = \frac{\alpha_S \Pi_S \alpha_R \Pi_R}{\alpha_S \Pi_S + \alpha_R \Pi_R}, \quad U = \frac{k\Pi}{G_S C_p^S}, \quad V = \frac{k\Pi}{G_R C_p^R},$$
$$\varepsilon_S = \frac{1}{G_S C_p^S}\left[q_S + \frac{\alpha_S \Pi_S}{\alpha_S \Pi_S + \alpha_R \Pi_R}\cdot q_w \right],$$
$$\varepsilon_R = \frac{1}{G_R C_p^R}\left[q_R + \frac{\alpha_R \Pi_R}{\alpha_S \Pi_S + \alpha_R \Pi_R} q_w \right]. \quad (5)$$

Eq. (4) takes the form

$$\frac{\partial T_S}{\partial x} - U(T_R - T_S) = -\varepsilon_S, \quad \frac{\partial T_R}{\partial x} - V(T_R - T_S) = \varepsilon_R. \quad (6)$$

The input temperatures of the flows are assumed to be specified

$$T_S|_{x=0} = T_1(\tau), \quad T_R|_{x=1} = T_3(\tau).$$

The nonlinear solution of the homogeneous equation (when q = 0) was studied in detail in [4, 5]. To determine the small quasidynamic corrections, consideration is limited here to the case when U, V = const (x); this allows a simple form of the basic temperature profiles parametrically depending on τ to be obtained from Eq. (6)

$$T_S(x) = T_1 + \frac{U\tilde{F}(x)}{D}(T_3 - T_1),$$
$$T_R(x) = T_3 + \frac{V[F - \tilde{F}(x)]}{D}(T_1 - T_3), \quad (7)$$

where

$$\tilde{F}(x) = [\exp(fx) - 1]/f, \quad F = \tilde{F}|_{x=1},$$
$$f = V - U, \quad D = 1 + VF. \quad (8)$$

The basic values of the input temperatures of the flows $T_2 = T_S|_{x=1}$, $T_4 = T_R|_{x=0}$ and the mean values $\bar{T}_S$, $\bar{T}_R$ required in what follows are determined from Eqs. (7) and (8)

$$T_2 = T_1 + \frac{UF}{D}(T_3 - T_1) \quad T_4 = T_3 + \frac{VF}{D}(T_1 - T_3), \quad (9)$$

$$\bar{T}_S = T_1 + \frac{U\bar{F}}{D}(T_3 - T_1), \quad \bar{T}_R = T_3 + \frac{V(F - \bar{F})}{D}(T_1 - T_3), \quad (10)$$

where $\bar{F} = (F - 1)/f$ (here and below, a bar over a symbol denotes coordinate averaging, for example: $\bar{T} = \int_0^1 T dx$).

To calculate the corrections to the output temperatures ΔT_2 and ΔT_4 due to inhomogeneity of Eq. (6), Eq. (6) is integrated with $\varepsilon \neq 0$ with the same boundary conditions. Then

$$\Delta T_2 = -\int_0^1 dx \left\{ \varepsilon_S \left[1 - \frac{U\tilde{F}(1-x)}{D} \right] + \varepsilon_R \frac{U}{D}\left[F - \tilde{F}(1-x) \right] \right\},$$
$$\Delta T_4 = -\int_0^1 dx \left\{ \varepsilon_S \frac{V\tilde{F}(1-x)}{D} + \varepsilon_R \frac{[1 + V\tilde{F}(1-x)]}{D} \right\}. \quad (11)$$

Note further that the values of ε determined from Eq. (5) are expediently written in the form $\varepsilon = \varepsilon^d + \hat{\varepsilon}$. It is clear from Eq. (4) that the dynamic component ε^d is related to the

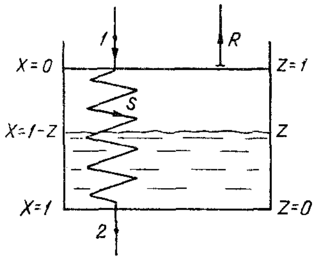

Fig. 2

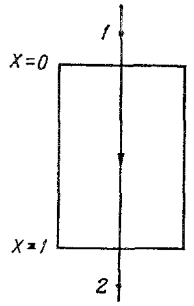

Fig. 3

Fig. 2. Indexing of boundary points and fluxes in immersion heat exchanger.

Fig. 3. Indexing of single-flow heat exchanger.

terms q proportional to $\partial/\partial\tau$, while $\hat{\varepsilon}$ characterizes the influence of the remaining factors included in $\hat{q}$. Correspondingly, the desired corrections may also be divided into components: $\Delta T = \Delta T^d + \Delta\hat{T}$.

In calculating $\Delta\hat{T}$, $\hat{\varepsilon}$ = const is adopted in Eq. (11), on the assumption that it is inexpedient to take account of the coordinate distribution of small $\hat{q}$; they may be averaged using Eq. (10). Then from Eq. (1)

$$\Delta\hat{T}_i = -\sum_{m=S,R} \sigma_{im}\hat{\varepsilon}_m, \tag{12}$$

where

$$\sigma_{2S} = 1 - \frac{U\bar{F}}{D}, \quad \sigma_{2R} = \frac{U(F-\bar{F})}{D}, \quad \sigma_{4S} = \frac{V\bar{F}}{D};$$
$$\sigma_{4R} = 1 - \frac{V(F-\bar{F})}{D}. \tag{13}$$

Giving specific form to the dynamic component ε^d in Eq. (5) in order to calculate ΔT^d, it must be noted that, according to Eq. (4), the part of ε^d which is proportional to $\partial T_w/\partial\tau$ contains the factor τ_w defined below. As may readily be shown, within the framework of the quasidynamic approximation

$$\tau_w \frac{\partial T_w}{\partial\tau} \approx \tau_w \frac{\partial\tilde{T}_w}{\partial\tau},$$

where $\tilde{T}_w = (\alpha_S\Pi_S T_S + \alpha_R\Pi_R T_R)/(\alpha_S\Pi_S + \alpha_R\Pi_R)$ is the quasi-equilibrium wall temperature. In fact, omitting the small quantity $\hat{q}_w$ in the third relation in Eq. (4), it is rewritten in the form

$$T_w = \tilde{T}_w - \tau_w \frac{\partial T_w}{\partial\tau},$$

which shows, after differentiation and discarding terms of order $(\tau_w\partial/\partial\tau)^2$, that the above statement is valid. In addition, it is clear from this relation that the initial disequilibrium of the wall temperature field is exponentially damped, with a relaxation period τ_w, so that times τ exceeding τ_w are considered below. As a result, it follows from Eqs. (4) and (5) that

$$\varepsilon_S^d = \tau_S \frac{\partial T_S}{\partial\tau} + \tau_w U \frac{\partial T_R}{\partial\tau},$$
$$\varepsilon_R^d = \tau_w V \frac{\partial T_S}{\partial\tau} + \tau_R \frac{\partial T_R}{\partial\tau}, \tag{14}$$

where

$$\tau_S = \frac{\rho_S\Omega_S}{G_S} + \tau_w \frac{(\alpha_S\Pi_S)^2}{G_S C_p^S(\alpha_S\Pi_S + \alpha_R\Pi_R)}, \tag{15}$$

$$\tau_R = \frac{\rho_R \Omega_R}{G_R} + \tau_w \frac{(\alpha_R \Pi_R)^2}{G_R C_p^R (\alpha_S \Pi_S + \alpha_R \Pi_R)},$$

$$\tau_w = \frac{\rho_w \Omega_w C_w}{(\alpha_S \Pi_S + \alpha_R \Pi_R)}.$$

To improve the quasidynamic description, it is now sufficient to substitute Eq. (14) into Eq. (11), using the basic profiles in Eq. (7) in calculating $(\partial T_S/\partial\tau)$ and $(\partial T_R/\partial\tau)$. Then the corrections $\Delta T_2{}^d$ and $\Delta T_4{}^d$ obtained are proportional to the time derivatives of the set of parameters (T_1, T_3, U, V) defining the profile in Eq. (7). The terms proportional to $\dot{T}_{1,3}$ may be written in compact form here, but calculation of the terms $\sim \dot{U}$ and $\dot{V}$ leads to extremely unwieldy final expressions, requiring additional simplifications. Taking the relatively weak dependence of U, V on the input parameters into account here, it is expedient to determine the coefficients of the expansion of ΔT^d with respect to U and V, assuming in Eq. (14) that

$$\varepsilon_m^d = \sum_{n=S,R} \tau_{mn} \frac{\partial \bar{T}_n}{\partial \tau} = \text{const}\,(x) \quad (m = S,\ R;\ \tau_{SS} = \tau_S;$$

$$\tau_{SR} = \tau_w U;\ \tau_{RS} = \tau_w V;\ \tau_{RR} = \tau_R).$$

Then, denoting $a_{ij} = \partial T_i/\partial T_j$ ($i = 2, 4$; $j = 1, 3$; $a_{21} = 1 - UF/D$; $a_{23} = UF/D$; $a_{41} = VF/D$; $a_{43} = 1 - VF/D$ according to Eq. (9), it is found from Eqs. (11)-(13) that

$$\Delta T_i^d = -\sum_{j=1,3} a_{ij}\tau_{ij} \frac{\partial T_j}{\partial \tau} - \sum_{m,n=S,R} \sigma_{im}\tau_{mn} \left.\frac{\partial \bar{T}_n}{\partial \tau}\right|_{T_{1,3}=\text{const}(\tau)} \tag{16a}$$

(τ_{ij} are given below). To use delayed arguments, $a_{i\omega} = \partial T_i/\partial\omega$ (ω = U, V) is introduced from Eq. (9). Then, on the basis of Eq. (3b), an equation equivalent to Eq. (16a) is obtained

$$\Delta T_i^d = -\sum_{j=1,3} a_{ij}\,[T_j(\tau) - T_j(\tau - \tau_{ij})] - \sum_{\omega=U,V} a_{i\omega}\,[\omega(\tau) - \omega(\tau - \tau_{i\omega})]. \tag{16b}$$

Then, letting $\bar{\bar{F}} = (F - 2\cdot\bar{F})/f$, it is found that

$$\tau_{21} - \tau_S = \tau_{43} - \tau_R = \frac{UV}{D}\,[(\tau_S + \tau_R)\bar{\bar{F}} + \tau_w 2(\bar{F} + V\bar{\bar{F}}), \tag{17a}$$

$$\tau_{23} = \tau_{41} = (\tau_S + \tau_R)\left(\frac{\bar{F}}{F} - \frac{U\bar{\bar{F}}}{DF}\right) + \tau_w\left[1 + \frac{2UV}{D}\left(\bar{F} - \frac{\bar{\bar{F}}}{F}\right)\right],$$

$$\tau_{i\omega} = \frac{1}{a_{i\omega}} \sum_{m,n=S,R} \sigma_{im}\tau_{mn} \frac{\partial \bar{T}_n}{\partial \omega}. \tag{17b}$$

As a result of these calculations, the relations obtained in Eq. (16) and the delay time in Eq. (17) describe the quasidynamic shift in output temperatures of a two-flow heat exchanger. Note that the error in the quasidynamic calculation of the output temperatures $T_2(\tau)$ and $T_4(\tau)$ is determined by the terms $(\tau_{delay}\partial T_{1,3}/\partial\tau)^2$, reaching values of 0.1-1% in all stages of normal cooling of cryogenic units.

The quasidynamic processes in three-flow, immersion, and one-flow heat exchangers may be described analogously. The relations determining the quasidynamics of the three-flow heat exchanger will be given in a subsequent work (on account of their unwieldiness). However, immersion and single-flow heat exchangers are characterized by sufficiently compact expressions, given below. Note that there is a certain generality in the description of all the elements, in particular, the unified system of differential equations in Eq. (18) for the interaction of the forward flow with the wall, assuming, as before, that the influence of the heat conduction and hydraulic losses is small, and the corresponding terms are symbolically included in $\hat{q}$

$$G_S C_p^S \frac{\partial T_S}{\partial x} - \alpha_S \Pi_S (T_w - T_S) = -q_S = -\left(\rho_S \Omega_S C_p^S \frac{\partial T_S}{\partial \tau} + \hat{q}_S\right),$$

$$\alpha_S \Pi_S (T_S - T_w) + \alpha_R \Pi_R (T_R - T_w) = q_w = \rho_w \Omega_w C_w \frac{\partial T_w}{\partial \tau} + \hat{q}_w. \quad (18)$$

In the case of an immersion heat exchanger, T_R is the temperature of the liquid determined by the saturated vapor pressure. Here x varies from (1 − Z) to 1, where Z (Fig. 2) characterizes the position of the liquid level at time τ (when Z = 1, the whole volume is filled with liquid and, when Z = 0, with gas). For a single-flow heat exchanger (Fig. 3), T_R is the temperature of the surrounding medium, and Z = 1.

Let

$$U = \frac{\alpha_S \Pi_S \alpha_R \Pi_R}{G_S C_p^S (\alpha_S \Pi_S + \alpha_R \Pi_R)}, \quad \varepsilon = \frac{1}{G_S C_p^S}\left[q_S + \frac{\alpha_S \Pi_S q_w}{(\alpha_S \Pi_S + \alpha_R \Pi_R)}\right]. \quad (19)$$

For the static case ($q_{S,w} = 0$), a nonlinear analytical solution taking account of the sharp dependence of α_R on T_w characteristic of a boiling liquid was investigated in [5]. Here, as in the previous model, U = const(x) is assumed in determining the quasidynamic corrections. Then, specifying the input temperature $T_S|_{x=1-Z} = T_S|_{x=0} = T^1(\tau)$ parametrically depending on τ, it is simple to find the basic output temperature T_2 and the integral mean temperature $\bar{T}_S$, which are required for the averaging of U, from Eq. (18) when q = 0

$$T_2 = T_R + (T_1 - T_R)\exp(f), \quad (20)$$

$$\bar{T}_S = T_R + (T_1 - T_R) F, \quad (21)$$

where F = (exp(f) − 1)/f, f = − UZ.

To determine the terms $\Delta\hat{T}_2$ related to the part of $\hat{\varepsilon}$ which is proportional to $\hat{q}$, it is assumed that $\hat{\varepsilon}$ = const, presuming the averaging of small terms $\hat{q}$ in Eq. (18) using the mean temperature from Eq, (21). Then

$$\Delta\hat{T}_2 = -ZF\hat{\varepsilon}. \quad (22)$$

Giving specific form to the dynamic part of ε^d, which is proportional to ∂/∂τ, the initial disequilibrium of the wall will be ignored here, as before, i.e., times τ exceeding τ_w are considered. Then, within the framework of the quasidynamic approximation, introducing the following terms according to Eq. (20)

$$a_\omega = \frac{\partial T_2}{\partial \omega} \ (\omega = T_1,\ T_R,\ U,\ Z;\ a_{T_1} = e^f;\ a_{T_R} = 1 - e^f;$$
$$a_U = -Z(T_1 - T_R)e^f;\ a_Z = -U(T_1 - T_R)e^f),$$

the quasidynamic shift in output temperatures may be obtained

$$\Delta T_2^d = -\sum_\omega a_\omega \tau_\omega \frac{\partial \omega}{\partial \tau} = -\sum_\omega a_\omega [\omega(\tau) - \omega(\tau - \tau_\omega)]. \quad (23)$$

The delay times τ_1, τ_R, τ_U, τ_Z of the parameters T_1, T_R, U, Z are given here by the expressions

$$\tau_1 = \tau_z = 2\tau_U = Z\tau_S, \quad \tau_R = \tau_w + Z\tau_S\left(1 - \frac{\bar{F}}{F}\right),$$

where

$$\tau_S = \frac{\rho_S \Omega_S}{G_S} + \tau_w \frac{(\alpha_S \Pi_S)^2}{G_S C_p^S (\alpha_S \Pi_S + \alpha_R \Pi_R)}; \quad \tau_w = \frac{\rho_w \Omega_w C_w}{(\alpha_S \Pi_S + \alpha_R \Pi_R)}; \quad (24)$$
$$\bar{F} = (F - 1)/f.$$

Note that the presence of the delayed argument in T_R takes account of possible temperature variation of the saturated vapors in an immersion heat exchanger or the temperature variation of the surrounding medium in a single-flow heat exchanger. For the latter, $Z(\tau) \equiv 1$, so that in this case τ_Z may be excluded from consideration.

NOTATION

τ, time, sec; x, normalized coordinate; T, temperature, K; C_p, isobaric specific heat, J/kg·K; G, heat-carrier flow rate, kg/sec; ρ, density, kg/m^3; Ω, pipeline volumes, m^3; α, heat-transfer coefficients, W/m^2·K; Π, heat-transfer surface, m^2; q, effective heat fluxes, W. Indices: S, forward flow; R, reverse flow; w, dividing wall.

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SOLUTION OF INVERSE PROBLEMS FOR A SYSTEM OF QUASILINEAR EQUATIONS OF HEAT CONDUCTION IN A SELF-SIMILAR REGIME

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Explicit solutions are obtained for inverse problems for a system of heat-conduction equations in a self-similar regime. The thermophysical characteristic being sought depend on the temperature distribution.

Mathematical modeling of a stationary heat-exchange process in two semiinfinite rods of different materials, joined by an "ideal" contact, is closely connected with the solution of an inverse problem concerned with the determination of the coefficients in the following system of nonlinear differential equations:

$$C_n(T_n)\,\frac{\partial T_n}{\partial t} = \frac{\partial}{\partial x}\left[\lambda_n(T_n)\,\frac{\partial T_n}{\partial x}\right] + \sum_{i=1}^{M_n} q_{in} t^{-3/2}\delta(x - x_i), \quad (x,\ t) \in \Omega_n, \tag{1}$$

with initial and boundary conditions

$$T_1(x,\ 0) = u_1, \quad x < 0, \quad T_2(x,\ 0) = u_2, \quad x > 0, \tag{2}$$

$$T_1(0,\ t) = T_2(0,\ t); \quad \lambda_1(T_1)\,\frac{\partial T_1}{\partial x}\bigg|_{x=0} = \lambda_2(T_2)\,\frac{\partial T_2}{\partial x}\bigg|_{x=0}, \quad t > 0, \tag{3}$$

where u_n are given constants, n = 1, 2.

The system (1)-(3) admits a self-similar solution of the form $T_n(x, t) = v_n(z)$, where $z = xt^{-1/2}$ and the function $v_n(z)$ satisfy the equations

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